2/27/17 "Mistakes are the portal of Discovery." - James Joyce

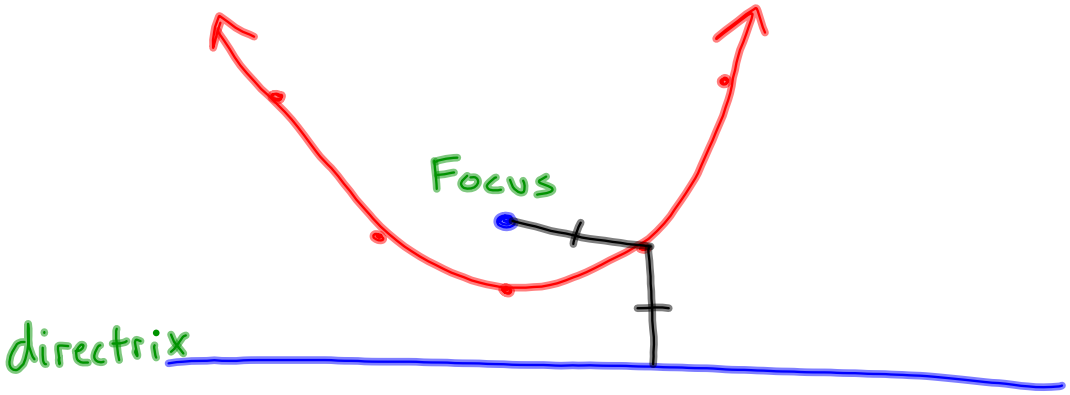
HW: "Locus Definition of a Parabola" Homework section

AIM: What is a Parabola?

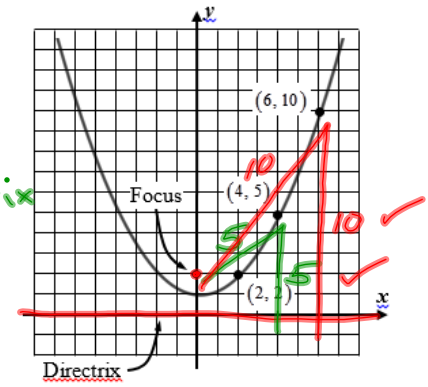
Warm Up:

THE LOCUS DEFINITION OF A PARABOLA

A parabola is the collection of all points **equidistant** from a fixed point (known as its **focus**) and a fixed line (known as its **directrix**).



***Exercise* #1:** The parabola $y = \frac{1}{4}x^2 + 1$ is shown graphed below with selected points shown. For this parabola, its focus is the point $(0, 2)$ and its directrix is the x-axis.



(a) How far is the turning point $(0, 1)$ from both the focus and directrix? How far is the point $(2, 2)$ from both?

Turning point is 1 unit away from focus and directrix

(2,2) is 2 units away from the focus and directrix.

(b) Use the distance formula to verify that the point $(4, 5)$ is the same distance away from the focus and directrix. Draw line segments from the focus and directrix to this point to visualize the distance. Repeat for the point $(6, 10)$

(0,2) and (4,5)

$d = \sqrt{(0-4)^2+(2-5)^2}$

$d = \sqrt{16+9}$

$d = \sqrt{25}$

$d = 5$

(0,2) and (6,10)

$d = \sqrt{(0-6)^2+(2-10)^2}$

$d = \sqrt{36+64}$

$d = \sqrt{100}$

$d = 10$

(c) Use the distance formula to show that the equation of this parabola is $y = \frac{1}{4}x^2 + 1$ based on the locus definition of a parabola.

(x, y)

Focus (0,2)

$d = \sqrt{(x-0)^2+(y-2)^2}$

Distance from any point (x,y) on parabola to the directrix is the y-value

y

$y = \sqrt{(x-0)^2+(y-2)^2}$

$y^2 = (x-0)^2+(y-2)^2$

$(y-2)^2 = (y-2)(y-2)$

$= y^2-4y+4$

$y^2 = x^2 + y^2 - 4y + 4$

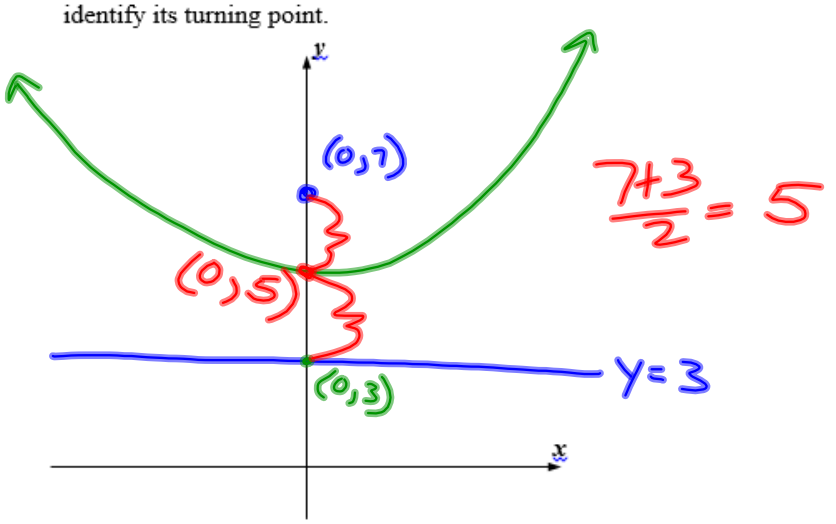
$0 = x^2 - 4y + 4$

$\frac{4y}{4} = \frac{x^2+4}{4}$

$y = \frac{1}{4}x^2 + 1$ ✓

Exercise #2: Consider a parabola whose focus is the point $(0, 7)$ and whose directrix is the line $y = 3$.

(a) Sketch a diagram of the parabola below and identify its turning point.



$\frac{7+3}{2} = 5$

(b) Determine the equation of the parabola using the locus definition.

(x, y) $(0, 7)$ (x, y) $y = 3$

$d = \sqrt{(x-0)^2+(y-7)^2}$ distance $y - 3$

$$(y-3)^2 = \left(\sqrt{(x-0)^2+(y-7)^2}\right)^2$$

$$y^2 - 6y + 9 = x^2 + (y-7)^2$$

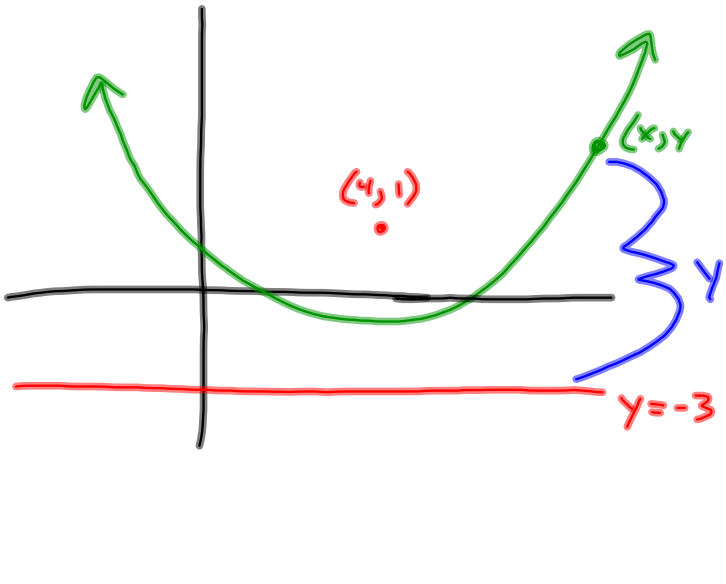
$$y^2 - 6y + 9 = x^2 + y^2 - 14y + 49$$

$+14y \quad -9 \qquad +14y \quad -9$

$$8y = x^2 + 40$$

$$y = \frac{1}{8}x^2 + 5$$

***Exercise* #3:** Determine the equation of the parabola whose focus is the point $(4, 1)$ and whose directrix is the horizontal line $y = -3$. First, draw a diagram that shows the parabola, then carefully use the distance formula to derive its equation.



$$y+3=\sqrt{(x-4)^2+(y-1)^2}$$

$$y^2+6y+9=(x-4)^2+(y-1)^2$$

$$y^2+6y+9=x^2-8x+16+y^2-2y+1$$

$$+2y-9 \qquad +2y-9$$

$$8y=x^2-8x+8$$

$$y=\frac{1}{8}x^2-x+1$$